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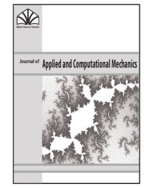

An Analytical Technique for Solving Nonlinear Oscillators of the Motion of a Rigid Rod Rocking Bock and Tapered Beams

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Abstract

In this paper, a new analytical approach has been presented for solving strongly nonlinear oscillator problems. Iteration perturbation method leads us to high accurate solution. Two different high nonlinear examples are also presented to show the application and accuracy of the presented method. The results are compared with analytical methods and with the numerical solution using Runge-Kutta method in different figures. It has been shown that the iteration perturbation approach doesn't need any small perturbation and is accurate for nonlinear oscillator equations.

Keywords: Periodic solution; Nonlinear oscillators; Motion of a rigid rod rocking back; Tapered beams.

1. Introduction

The solution of differential equations in physics and engineering, especially some oscillation equations are nonlinear, and in most cases it is difficult to solve such equations, especially analytically. Recently, several scientific papers were devoted to approximate analytical approximate solutions for nonlinear oscillators. Some approximate approaches have proposed to solve strongly nonlinear differential equations such as homotopy perturbation method [1-3], energy balance method [4-6], frequency amplitude formulation [7, 8], parameter expansion method [3, 9], variational iteration method [10, 11], max min approach [12, 13] hamiltonian approach [14-16], variational approach [17, 18], and other new methods [19-27].

The main propose of this paper is to obtain highly accurate analytical solution for free vibrations of strongly nonlinear oscillators. The iteration method solution has been compared with others method and numerical solution using Runge-Kutta method of order four. The results will show its effective and convenient approximate solution.

The paper has been organized as follows. In Section 2, we present the analytical procedure. In Section 3, we applied iteration procedure for solving two important applications. Section 4 provides some comparisons between analytical and numerical solutions. In conclusion, in the last section the most important findings of the paper have been presented.

2. Solution Procedure

We consider a generalized nonlinear oscillator in the form:

$$\ddot{u} + f\,(u\,,\,\dot{u}\,,\,\ddot{u}) = 0, \tag{1}$$

with initial conditions:

$$u(0) = A, \qquad \dot{u}(0) = 0. \tag{2}$$

Based on He's frequency-amplitude formulation approach [28, 29]. The trial function to determine the angular frequency ω is given by

$$u = A\cos\omega t. \tag{3}$$

Substituting from Eq. (3) into Eq. (1), one can obtain the following residual as

$$R(t) = -A\omega^2\cos\omega t + f(A\cos\omega t, -A\omega\sin\omega t, -A\omega^2\cos\omega t). \tag{4}$$

Introducing a new function, $H(t)$, defined as [30]

$$H(t) = \int_0^T R(t)\cos(\omega t)dt = 0, \qquad T = \frac{2\pi}{\omega}. \tag{5}$$

Solving the above equation, the relationship between the frequency and amplitude of the oscillator can be obtained.

3. Applications

In order to assess the advantages and the accuracy of the iteration procedure, the following two examples are considered.

3.1 The Motion of a Rigid Rod Rocking Back

The motion of a rigid rod rocking back and forth on the circular surface without slipping. The governing equation of motion can be expressed as [31-34].

$$\left(\frac{1}{12} + \frac{1}{16}u^2\right)\frac{d^2u}{dt^2} + \frac{1}{16}u\left(\frac{du}{dt}\right)^2 + \frac{g}{4L}u\cos u = 0, \quad u(0) = A, \quad \frac{du}{dt}(0) = 0, \tag{6}$$

where $g > 0, L > 0$ are known positive constants.

By using the following trial function to determine the angular frequency ω:

$$u = A\cos\omega t. \tag{7}$$

Substituting Eq. (7) into Eq. (6) results in, the following residual

$$\begin{aligned} R(t) = \tfrac{A}{1536L}\Big[&\left(384g - 144A^2g + 10A^4g - 128L\omega^2 - 48L\omega^2A^2\right)\cos\omega t \\ &-\left(48A^2g - 5A^4g + 48A^2L\omega^2\right)\cos 3\omega t + A^4g\cos 5\omega t.\Big] \end{aligned} \tag{8}$$

Using Eq. (8) into Eq. (5), we can easily obtain

$$H(t) = \int_0^{2\pi/\omega} R(t)\cos\omega t dt = \frac{A\pi}{768L\omega}\left[(192 - 72A^2 + 5A^4)g - 8(8 + 3A^2)L\omega^2\right] = 0. \tag{9}$$

Solving the above equation, an approximate frequency ω as a function of amplitude A as follow:

$$\omega = \frac{\sqrt{\left(192 - 72A^2 + 5A^4\right)g}}{\sqrt{8L(8 + 3A^2)}}. \tag{10}$$

Hence, the approximate solution can be readily obtained

$$u(t) = A\cos\left(\frac{\sqrt{\left(192 - 72A^2 + 5A^4\right)g}}{\sqrt{8L(8+3A^2)}}t\right). \tag{11}$$

3.2 Tapered Beams

Tapered beams can model engineering structures which require a variable stiffness along the length, such as moving arms and turbine blades. In dimensionless form, the governing differential equation corresponding to fundamental vibration mode of tapered beams is given by [35, 36]. As we see in the geometry of problem in Fig. 1, m_1 is mass of the block on the horizontal surface, m_2 is the mass of block which is just slipped in the vertical and is linked to m_1, L is length of link, g is gravitational acceleration, and k is spring constant [37, 38].

By assuming $u = \frac{x}{L}, |u| \ll 1$, the equation of motion can be yield as following terms:

$$\left(\frac{d^2u}{dt^2}\right) + \left(\frac{m_2}{m_1}\right)u^2\left(\frac{d^2u}{dt^2}\right) + \left(\frac{m_2}{m_1}\right)u\left(\frac{du}{dt}\right)^2 + \left(\frac{k}{m_1} + \frac{m_2 g}{Lm_1}\right)\mathrm{u} + \left(\frac{m_2 g}{2Lm_1}\right)u^3 = 0. \tag{12}$$

The initial conditions for Eq. (12) are given by $u(0) = A$ and $\dot{u}(0) = 0$. Here u and t are generalized dimensionless displacement and time variables.

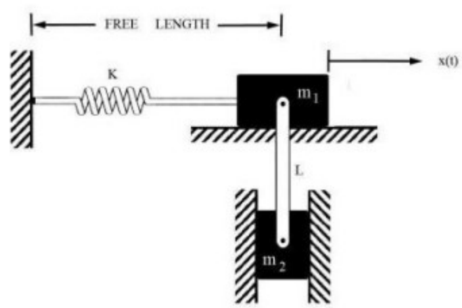


Fig. 1 Geometric of the tapered beams

The use of Eqs. (3-5), and (12) leads to the relationship between amplitude and angular frequency.

$$\omega = \sqrt{\frac{8kL + 8gm_2 + 3A^2 gm_2}{8m_1L + 4A^2 m_2 L}}. \tag{13}$$

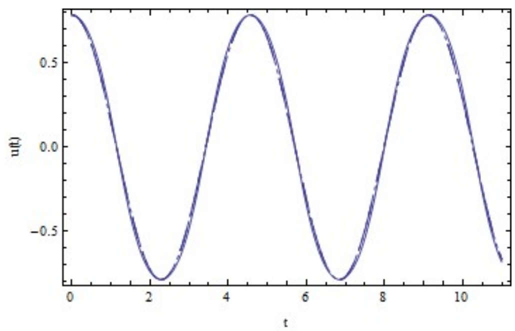
Hence, the approximate solution can be readily obtained as:

$$u(t) = A\cos\left(\sqrt{\frac{8kL + 8gm_2 + 3A^2 gm_2}{8m_1L + 4A^2 m_2 L}}t\right). \tag{14}$$

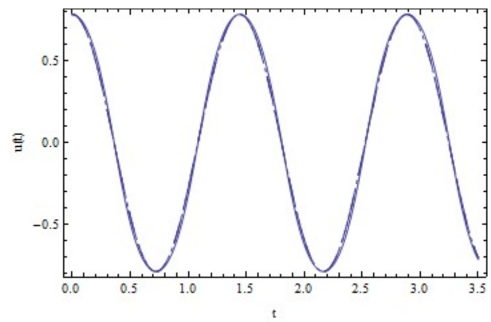
4. Results and Dissection

In this section, an approximate technique is developed based on He's frequency-amplitude formulation and He's energy balance method to solve strongly nonlinear differential equations. The solutions for two nonlinear problems show a good agreement with the numerical solutions using Runge-Kutta method.

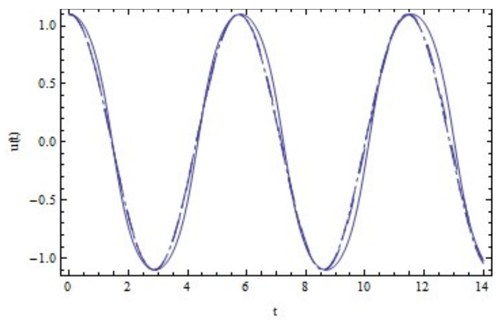
In Fig. 2 the comparison between Analytical solutions and Runge-Kutta method is shown. As we see, the results are compared with amplitude frequency formulation [32], energy balance method [33, 34] and with an accurate numerical solution, using fourth order Runge-Kutta method to show the accuracy of the method. It has been indicated that the present method has excellent agreement with the numerical solution. It is a simple method and easy to apply to any kind of nonlinear vibration problems.



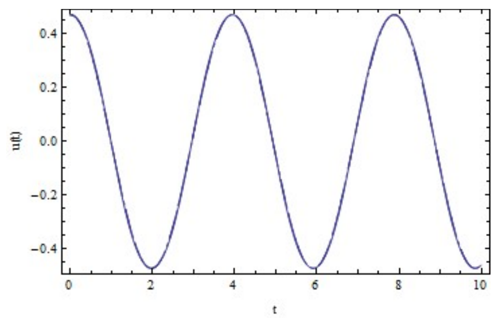
$A = 0.25\pi, g = 1, L = 1$



$A = 0.25\pi, g = 10, L = 1$



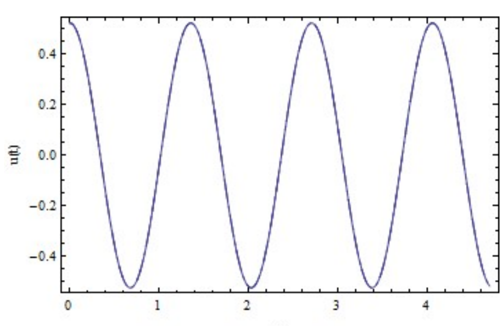
$A = 0.35\pi, g = 1, L = 1$



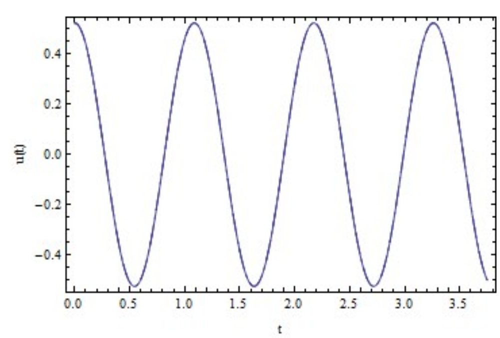
$A = 0.15\pi, g = 1, L = 1$

Fig 2. The comparison between analytical solution (….), energy balance method (- - -), amplitude frequency formulation (── ──) and numerical solution (──).

Fig. 3 represents the comparison of the analytical solution with the numerical solution for different parameters in two cases to show the accuracy of the method. It has been shown that the results of analytical approximate solution is the same with those obtained from the results of the max-min approach [37], amplitude frequency formulation [37, 38], energy balance method [38], and have a high validity in comparison with the numerical solution using fourth order Runge-Kutta method.



$A = \frac{\pi}{6}, g = 9.81, k = 100, m_1 = 5, m_2 = 1, L = 1$



$A = \frac{\pi}{6}, g = 9.81, k = 300, m_1 = 10, m_2 = 2, L = 0.5$

Fig 3. The comparison between analytical solutions (.......) and numerical solution (──).

5. Conclusion

Based on He’s frequency-amplitude formulation and He’s energy balance method, a new analytical technique has been presented to determine approximate solutions of some strongly nonlinear differential equations. In compared with the previously published methods, determination of solutions is straightforward and simple. In comparison to forth-order Runge-Kutta method, which is powerful numerical solution, the results show that the present method is very convenient for solving nonlinear equations and also can be used for strong nonlinear oscillators.

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